UNIVERSIDAD Finis Terrae

VINCE IN BONO MALUM

# Guía 9
# Operaciones y composición de Funciones

1. Sean las funciones $f(x)=\dfrac{2x-1}{x+1}$, $g(x)=\dfrac{x}{x-1}$. Determine la expresión funcional indicando dominio:
   a. $(f+g)(x)$
   b. $(f-g)(x)$
   c. $(f\cdot g)(x)$
   d. $\left(\dfrac{f}{g}\right)(x)$

2. Sea $f(x)=2x^2+3x+4$, calcule:
   a. $\dfrac{f(x+h)-f(x)}{h}$
   b. $\dfrac{f(x)-f(3)}{x-3}$

3. Sean las funciones $f(x)=\dfrac{2x-1}{x+1}$, $g(x)=\dfrac{x}{x-1}$, calcule:
   a. $(g \circ f)(x)$
   b. $(f \circ g)(x)$

4. Dada las funciones $f(x)=\dfrac{x}{x-1}$, $g(x)=1-\dfrac{1}{x}$, y $h(x)=\dfrac{1}{x}$. Verificar que:

$$\left[(h \circ g)\circ f\right](x)=\left[h\circ(g\circ f)\right](x)$$

5. Determine (fog)(x) y (gof)(x), en los siguientes ejercicios:
   a. $f(x)=\sqrt{x}; g(x)=x^2$
   b. $f(x)=\dfrac{1}{x+1}; g(x)=\sqrt{x}+1$
   c. $f(x)=2+\sqrt{x}; g(x)=(x-2)^2$
   d. $f(x)=x^2+2; g(x)=x-3$
   e. $f(x)=x^2; g(x)=\sqrt{x}+1$
   f. $f(x)=|x|; g(x)=x^2$
   g. $f(x)=\dfrac{1}{x}; g(x)=x-1$
   h. $f(x)=\dfrac{x+1}{3}; g(x)=3x-1$
   i. $f(x)=3; g(x)=7$
   j. $f(x)=4; g(x)=x^2$
   k. $f(x)=\dfrac{1}{x+1}; g(x)=2x^2-3x+1$

6. Calcule la suma, diferencia, el producto y el cuociente de las dos funciones f y g en cada uno de los siguientes ejercicios. Determine los dominios de las funciones resultantes.
   a. $f(x)=x^2; g(x)=\dfrac{1}{x-1}$
   b. $f(x)=x^2+1; g(x)=\sqrt{x}$
   c. $f(x)=\sqrt{x-1}; g(x)=\dfrac{1}{x+2}$
   d. $f(x)=1+\sqrt{x}; g(x)=\dfrac{2x+1}{x+2}$
   e. $f(x)=(x+1)^2; g(x)=\dfrac{1}{x^2-1}$

Respuestas:

1. Sean las funciones $f(x)=\frac{2x-1}{x+1}$, $g(x)=\frac{x}{x-1}$. Determine la expresión funcional indicando dominio:

$$Dom\, f(x)=IR-\{-1\},\quad Dom\, g(x)=IR-\{1\}$$

a. $(f+g)(x)=\frac{3x^2-2x+1}{(x+1)(x-1)}$

b. $(f-g)(x)=\frac{x^2-4x+1}{(x+1)(x-1)}$

c. $(f\cdot g)(x)=\frac{(2x-1)x}{(x+1)(x-1)}$

d. $\left(\frac{f}{g}\right)(x)=\frac{(2x-1)(x-1)}{(x+1)\,x}$

Por lo que el dominio de $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$ es $IR-\{-1,1\}$

Y el dominio de $\left(\frac{f}{g}\right)(x)$ es $IR-\{-1,1,0\}$

2.

a. $$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)^2+3(x+h)+4-2x^2-3x-4}{h}$$
$$=\frac{2x^2+4xh+2h^2+3x+3h+4-2x^2-3x-4}{h}$$
$$=\frac{4xh+2h^2+3h}{h}=\frac{h(4x+2h+3)}{h}=4x+2h+3$$

b. $$\frac{f(x)-f(3)}{x-3}=\frac{2x^2+3x+4-(2\cdot 9+9+4)}{x-3}=\frac{2x^2+3x-27}{x-3}$$
$$=\frac{(2x+9)(x-3)}{x-3}=2x+9$$

3.

a. $(g \quad f)(x) = g(f(x)) = g\left(\frac{2x-1}{x+1}\right) = \frac{\frac{2x-1}{x+1}}{\frac{2x-1}{x+1}-1} = \frac{2x-1}{x+2}$

$$Dom(gof) = IR - \{2\}$$

b. $(f \quad g)(x) = f(g(x)) = f\left(\frac{x}{x-1}\right) = \frac{2\frac{x}{x-1}-1}{\frac{x}{x-1}+1} = \frac{x+1}{2x-1}$

$$Dom(fog) = IR - \{1/2\}$$

4. $f(x) = \frac{x}{x-1}$, $g(x) = 1-\frac{1}{x}$, y $h(x) = \frac{1}{x}$. Verificar que:

$$[(h \quad g) \quad f](x) = ((h(g(x)))of) = Tof(x) = T(f(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = x$$

$$h(g(x)) = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1} = T(x)$$

$$[h \quad (g \quad f)](x) = h(g(f(x)) = h\left(1-\frac{1}{\frac{x}{x-1}}\right) = h\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$$

Por lo tanto son equivalentes.

5.

a. $fog(x) = |x|$ ; $gof(x) = x$

b. $fog(x) = \frac{1}{\sqrt{x}+2}$ ; $gof(x) = \sqrt{\frac{1}{x+1}}+1$

c. $fog(x) = 2+|x-2|$ ; $gof(x) = x$

d. $fog(x) = (x-3)^2$ ; $gof(x) = x^2-1$

e. $fog(x) = \left(\sqrt{x}+1\right)^2$ ; $gof(x) = |x|+1$

f. $fog(x) = x^2$ ; $gof(x) = x^2$

g. $fog(x) = \frac{1}{x-1}$ ; $gof(x) = \frac{1}{x}-1$

h. $fog(x) = x$; $gof(x) = x$

i. $fog(x) = 3$ ; $gof(x) = 7$

j. $fog(x) = 4$ ; $gof(x) = 16$

k.
$$fog(x) = \frac{1}{2x^2 - 3x + 2};$$
$$gof(x) = \frac{x^2 - x}{(x+1)^2}$$

6.

a. $f(x) = x^2; g(x) = \dfrac{1}{x-1}$

$$(f+g)(x) = \frac{x^3 - x^2 + 1}{x-1}$$
$$(f-g)(x) = \frac{x^3 - x^2 - 1}{x-1}$$
$$(f \cdot g)(x) = \frac{x^2}{x-1}$$
$$Dom(f+g)(x) = Dom(f-g)(x) = Dom(f \cdot g)(x) = IR - \{1\}$$
$$\left(\frac{f}{g}\right)(x) = x^2(x-1)$$
$$Dom\left(\frac{f}{g}\right)(x) = IR - \{1\}$$

b. $f(x) = x^2 + 1; g(x) = \sqrt{x}$

$$(f+g)(x) = x^2 + 1 + \sqrt{x}$$
$$(f-g)(x) = x^2 + 1 - \sqrt{x}$$
$$(f \cdot g)(x) = (x^2 + 1)\sqrt{x}$$
$$Dom(f+g)(x) = Dom(f-g)(x) = Dom(f \cdot g)(x) = IR_0^+$$
$$\left(\frac{f}{g}\right)(x) = \frac{x^2+1}{\sqrt{x}}$$
$$Dom\left(\frac{f}{g}\right)(x) = IR^+$$

c. $f(x) = \sqrt{x-1}; g(x) = \dfrac{1}{x+2}$

$$(f+g)(x)=\sqrt{x-1}+\frac{1}{x+2}$$

$$(f-g)(x)=\sqrt{x-1}-\frac{1}{x+2}$$

$$(f\cdot g)(x)=\frac{\sqrt{x-1}}{x+2}$$

$$Dom(f+g)(x)=Dom(f-g)(x)=Dom(f\cdot g)(x)=[1,\infty+[$$

$$\left(\frac{f}{g}\right)(x)=\sqrt{x-1}(x+2)$$

$$Dom\left(\frac{f}{g}\right)(x)=[1,\infty+[$$

d. $f(x)=1+\sqrt{x};\, g(x)=\dfrac{2x+1}{x+2}$

$$(f+g)(x)=1+\sqrt{x}+\frac{2x+1}{x+2}$$

$$(f-g)(x)=1+\sqrt{x}-\frac{2x+1}{x+2}$$

$$(f\cdot g)(x)=\frac{(1+\sqrt{x})(2x+1)}{x+2}$$

$$Dom(f+g)(x)=Dom(f-g)(x)=Dom(f\cdot g)(x)=[0,\infty+[$$

$$\left(\frac{f}{g}\right)(x)=\frac{(1+\sqrt{x})(x+2)}{2x+1}$$

$$Dom\left(\frac{f}{g}\right)(x)=[0,\infty+[$$

e. $f(x)=(x+1)^2;\, g(x)=\dfrac{1}{x^2-1}$

$$(f+g)(x) = (x+1)^2 + \frac{1}{x^2-1}$$

$$(f-g)(x) = (x+1)^2 - \frac{1}{x^2-1}$$

$$(f\cdot g)(x) = \frac{(x+1)^2}{x^2-1}$$

$$Dom(f+g)(x) = Dom(f-g)(x) = Dom(f\cdot g)(x) = IR - \{-1,1\}$$

$$\left(\frac{f}{g}\right)(x) = (x+1)^2(x^2-1)$$

$$Dom\left(\frac{f}{g}\right)(x) = IR - \{-1,1\}$$